1. Which of the following shows the complete boundary conditions to the minimization problem, $\underset{y(x)}{\text{Min}}\ J=\int_{x_1}^{x_2} F(y(x),y'(x))dx$, at x_1 ?

(a) $\dfrac{\partial F}{\partial y'}=0$

(b) $\delta y=0$

(c) $\dfrac{\partial F}{\partial y'}=0$ or $\delta y=0$

(d) $\dfrac{\partial F}{\partial y'}=0$ and $\delta y=0$

2. Which of the following represents Euler-Lagrange equation for a functional containing up to the fourth derivative of a function?

(a) $F_y-(F_{y'})'-(F_{y''})''-(F_{y'''})'''-(F_{y''''})''''$

(b) $F_y-(F_{y'})'+(F_{y''})''-(F_{y'''})'''+(F_{y''''})''''$

(c) $F_{y'}-(F_{y'})'+(F_{y'})''-(F_{y'})'''+(F_{y'})''''$

(d) $F_y-(F_{y''''})''''$

3. Which of the following statements is true?

(a) A function, y(x) that satisfies the Euler-Lagrange equation and the imposed boundary conditions is by definition the minimizing function for the integral for which the necessary conditions were derived.

(b) A function, y(x) that satisfies the Euler-Lagrange equation and the imposed boundary conditions is by definition the maximizing function for the integral for which the necessary conditions were derived.

(c) A function, y(x) that satisfies the Euler-Lagrange equation and the imposed boundary conditions is by definition the extremizing function for the integral for which the necessary conditions were derived.

(d) All of the above.

4. If we minimize $J = \int_0^1 (y'^2 + y)dx$ with $y(0) = 2$ and $y(1): Free$, then $y(x) = ?$

(a) $\frac{x^2}{4} - \frac{x}{4} + 2$

(b) $\frac{x^2}{4} - \frac{x}{2} + 2$

(c) $\frac{5x^2}{6} - \frac{x}{6} + 2$

(d) $\frac{x^2}{6} - \frac{5x}{6} + 2$

5. Steps that can help you obtain Euler-Lagrange equations for integrands with multiple functions are given below.

(i) Use Integration by parts to reduce terms with variations of derivatives of the unknown function.

(ii) Separate out the boundary terms.

(iii) Take variation of the integral with respect to both the functions.

(iv) Invoke fundamental lemma on the integral term.

Identify the order in which these steps has to be implemented.

(a) iii-i-ii-iv

(b) iii-ii-i-iv

(c) i-iv-iii-ii

(d) i-ii-iii-iv

6. Choose the correct statement among the following.

(a) Local constraints pertain to the entire domain compared to global constraints that are imposed at every point in the domain.

(b) A global constraint can never be a differential equation.

(c) In finite variable optimization, we learned that KKT conditions are valid only when the constraint qualification is satisfied. But there is no such restriction in calculus of variations problems with constraints.

(d) An isoperimetric constraint is a local constraint.

7. Lagrange multiplier corresponding to a local constraint is a…

(a) Function

(b) Scalar

(c) Functional

(d) Any of the above

8. Which of the following is common to both finite variable optimization and calculus of variation?

(a) Complementarity conditions

(b) Constraint Qualification

(c) Concept of Lagrangian and Lagrange multiplier

(d) All of the above

9. How do we add the term corresponding to the constraint, $g(x, y(x), z(x)) = 0$, to write the Lagrangian of the problem?

(a) $g(x, y, z)$

(b) $\lambda(x) g(x, y(x), z(x))$

(c) $\int \lambda(x) g(x, y(x), z(x)) dx$

(d) $\lambda(x) \int g(x, y(x), z(x)) dx$

10. Optimization statement for designing the stiffest bar for a given volume can be written as :

$$\underset{A(x)}{Min}\ J = \int_0^L pudx$$

Subject to

$$\lambda : (EAu')' + p = 0$$

$$\Lambda : \int_0^L Adx \leq V^*$$

Select the correct Lagrangian, L for the above problem.

(a) $L = pu + \lambda(x)((EAu')' + p)$

(b) $L = \lambda(x)((EAu')' + p) + \Lambda\left(A - V^*\right)$

(c) $L = pu + \Lambda\left(A - V^*\right)$

(d) $L = pu + \lambda(x)((EAu')' + p) + \Lambda\left(A - V^*\right)$